

THE LAST RESORT: ESTIMATING SCORES WHEN BAR EXAM PARTS ARE MISSING

by Andrew A. Mroch, Ph.D.

Despite careful planning and preparation, problems can arise with bar exam administrations. One of these problems occurs when one or more parts of the bar exam are missing for one or more candidates.

Dealing with a missing part of the bar exam is the focus of this article, specifically the last resort in such situations: predicting a candidate's score on the missing part using the remaining exam parts. This predicted score can then be used as the basis for an estimated score and a pass/fail decision on the bar exam. Using a predicted score on the missing part should not be taken lightly. This is truly the last resort for obtaining a pass/fail decision when a candidate does not have scores on all parts of the bar examination and should be undertaken only after careful consideration of the situation.

Predicting a missing score always involves some amount of uncertainty in the predicted score; the candidate's actual performance (i.e., if the part had not been missing) could have resulted in a score that was higher or lower than the predicted score. The reasonableness and usefulness of a predicted score on the missing part of the bar exam depends on several factors, including the following:

- which part of the exam is missing,
- the proportion of the exam that is missing,

- the proportion of candidates affected,
- how the part was lost, and
- the candidate's performance on the other parts of the examination.

Reasonable prediction of a score on the missing part can usually be obtained when the proportion of the bar exam missing is small and when the proportion of candidates affected is small. For example, a single candidate (out of many) missing one essay (out of several) will allow for the best possible prediction of the missing part. As more parts of the bar exam are missing and/or more candidates are affected, we can be less certain about the predicted score and less certain about the estimated bar exam score, which ultimately can make the pass/fail decision less clear (and from a statistical perspective, less stable), particularly when the candidate's estimated bar exam score is near the passing score.

EXAMPLES OF MISSING BAR EXAM PARTS

The National Conference of Bar Examiners (NCBE) has provided assistance to jurisdictions when part or parts of the bar examination have been missing. Here are three examples of such cases: In one situation, a computer software problem led to one or more lost essay responses for a small number of candidates taking the essay component of the bar exam by computer. In another situation, a shipping problem led

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to one essay response being lost for a small number of candidates. In a third situation, a candidate did not follow protocol upon exam completion and the candidate's answer sheet was missing.

In each of these examples, a relatively small proportion of candidates was affected and, in most cases, a small proportion of the bar exam was affected. The circumstances in each of these examples were varied, but the process for obtaining a pass/fail decision when part of the bar exam is missing is similar. This process involves three possible steps, which are described next.

OBTAINING AN ESTIMATED PASS/FAIL DECISION

Step 1: Checking the Boundaries

The first step does not involve predicting a score on the missing part or estimating a score on the bar examination but involves checking to see if a clear pass/fail decision can be obtained without the missing part. This step is referred to as *checking the boundaries* and involves substituting first a score of zero and then a perfect score for the missing part and calculating the resulting bar exam score, using the scores on the non-missing parts.¹ Substituting these two scores for the score on the missing part provides us with the range of possible scores a candidate could have obtained on the bar exam, given his or her scores on the non-missing parts.

Checking the boundaries may provide an unambiguous pass/fail decision in two cases: not passing

with a perfect score on the missing part and passing with a score of zero on the missing part. These two cases represent clear pass/fail decisions that do not require predicting a score on the missing part. In both cases, the score on the missing part is not required to make a pass/fail decision on the bar exam; either scores are low enough that even perfect performance on the missing part would result in a failing score or scores are high enough that even a score of zero on the missing part would result in a passing score.

Checking the boundaries will, however, frequently result in an ambiguous pass/fail decision. In such a situation, the candidate will not pass with a score of zero on the missing part, but will pass with a perfect score on the missing part. When the pass/fail decision is not resolvable by checking the boundaries, we are confronted with a serious problem. This is where the last resort, predicting the score, becomes a possibility.

Step 2: Predicting the Score

In order to predict the score on the missing part, scores on the non-missing parts are used to develop a *linear regression* equation representing the *regression line* that predicts a candidate's score on the missing part from a weighted sum of the scores on the non-missing parts.² Linear regression analysis is typically used to identify an equation that yields the best prediction of a dependent variable (in this case, the missing part) based on one or more independent variables (in this case, the non-missing parts) in the sense that the average squared difference between

the predicted and observed values of the dependent variable is minimized.³ Using the scores of candidates with complete data (i.e., no missing parts), an equation is developed that provides the best prediction of a candidate's score on the missing part of the bar exam from his or her scores on the other parts. The missing score can then be predicted by substituting the candidate's non-missing scores into the equation to obtain his or her predicted score on the missing part.

Figure 1 shows a hypothetical regression line predicting an essay score from the remaining bar exam parts (typically, the MBE and remaining essay

scores).⁴ This example would apply to a situation where an essay response is missing for a candidate and the remaining parts of the exam are used to predict the missing essay score. The cloud of points in Figure 1 includes scores for candidates with complete information; each point represents a candidate's score on the bar exam without the essay (horizontal axis) and a candidate's score on the essay alone (the vertical axis). These points are used as the basis for determining the regression line; specifically, the regression line minimizes the average squared vertical difference between the cloud of points and the line.⁵ The regression equation that defines the line

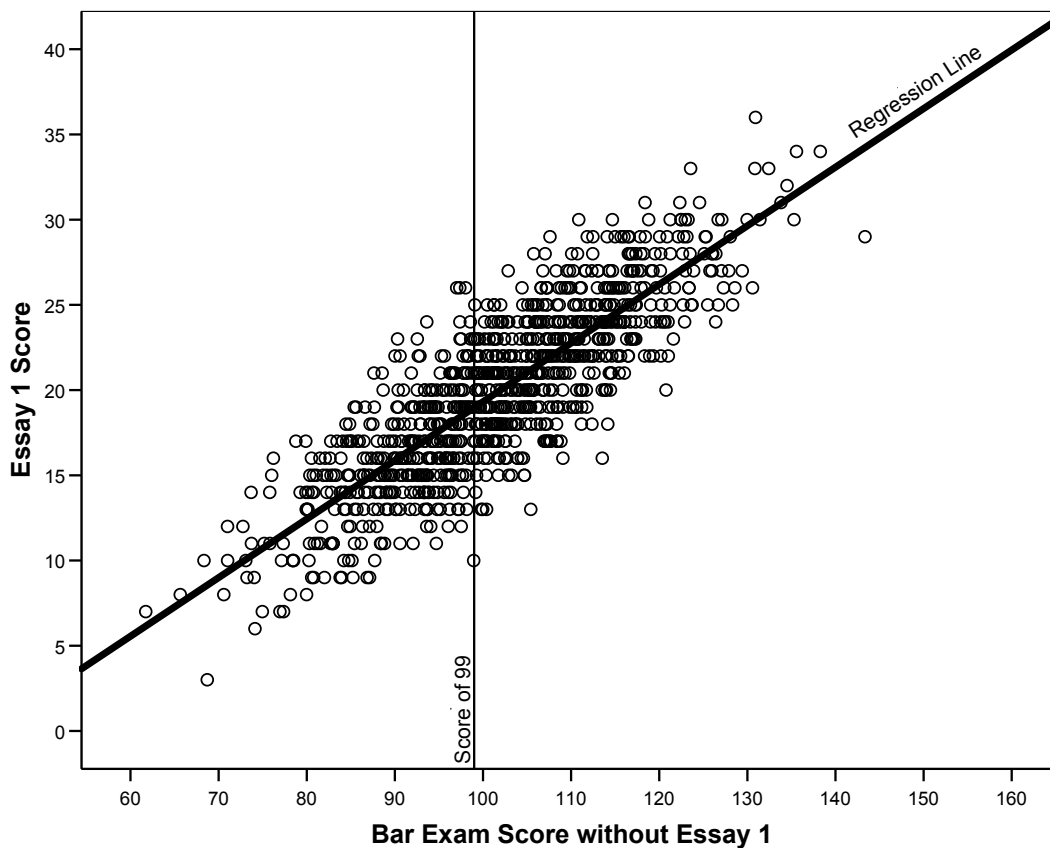


Figure 1: Scatterplot of hypothetical scores on the bar exam without the essay, and scores on the essay alone; regression line predicting an essay score from scores on the remainder of the bar exam

shown in Figure 1 can then be used to predict a person's score on the missing essay from his or her score on the remaining parts of the bar exam. For example, a candidate with a score of 99 on the remaining parts of the bar exam would have a predicted essay score of approximately 19.

The regression line and corresponding equation provide a statistically optimal estimate of a candidate's score on the missing part, given his or her score on the other parts. Such estimates are referred to as *point estimates* because they provide a single, specific estimate of each candidate's score on the score scale. From the predicted score on the missing part, an estimated bar exam score can be calculated. This estimated score can then be used to make a pass/fail decision, once a decision rule has been defined.

Step 3: Defining a Decision Rule

A *decision rule* defines the level of performance on the bar exam required to pass. When part of the bar exam is missing, a decision rule typically takes into account the uncertainty in the predicted score on the missing part. Decision rules can be defined by the probability that a candidate's score would be at or above the passing score if there were no missing information. For example, a decision rule of 50% will pass a candidate if there is a 50% chance that the candidate's estimated score (his or her bar exam score using the linear regression point estimate of the missing part) would be at or above the passing score. If the candidate's estimated score is at or above the passing score, the candidate will pass. A decision

rule of 10%, which is a lenient rule, will pass a candidate if there is a 10% chance that the candidate's score would be at or above the passing score. A decision rule of 90%, which is a stringent rule, will pass a candidate if there is a 90% chance that the candidate's score would be at or above the passing score.

The decision rule is a policy decision that is characterized by its degree of stringency or leniency. The amount of stringency or leniency can be thought of as the degree of "benefit of the doubt" given to a candidate whose bar exam score is subject to uncertainty because the score is estimated using a predicted score on the missing part. Additional details about decision rules and uncertainty in predicted scores are discussed in the next section.

A TECHNICAL DISCUSSION

OF REGRESSION ESTIMATES AND DECISION RULES

An estimate of the bar exam score obtained from the point estimate of the missing part can be used by itself to make a pass/fail decision. If the point estimate of the missing part results in an estimated bar exam score that is above the passing score, the candidate passes. If the point estimate results in an estimated bar exam score that is below the passing score, the candidate does not pass. This rule uses the point estimate as if it were the candidate's score. It resolves pass/fail decisions in a reasonable, evenhanded way, but it does not explicitly account for uncertainty in prediction. Candidates with point estimates just below the passing score would not

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pass under this rule, but almost half of these candidates would probably pass if their missing scores were known. Similarly, candidates with point estimates just above the passing score would pass, but almost half of these candidates would probably not pass if their missing scores were known.

Regression estimates of any variables that are used to make predictions are based on partial information (e.g., scores on the MBE and some essays) and are never perfect. There is always some spread, or variability, around the regression line, and for any estimated value of the dependent variable (i.e., the missing score), the actual values of the scores for candidates with that predicted value will generally not equal the predicted value. If the prediction is good, the actual values of the predicted variable will be expected to cluster around the regression line, but they will not generally be *on* the line. About half of the actual values will be above the line and about half of the actual values will be below the line. Most of the points will be close to the line, some will be farther away, and a few may be quite distant from the line, even if the relationship between the variables is quite strong.

Figure 1 can be used to illustrate the variation of candidate responses from the regression line for a hypothetical group of candidates taking a bar exam composed of the MBE and five essay questions.⁶ The points represent candidates' scores on one essay (presuming that the response to this essay question

is missing for some candidates) against their scores on the remainder of the exam. For example, if we pick a score of 99 on the horizontal axis (representing the bar exam score without one essay score), which is shown by the vertical line in the figure, there are a range of observed scores on the essay; most essay scores are near the regression line, between about 15 and about 23, but a few are farther from the line. One candidate had an essay score of about 10. The predicted score on the essay (i.e., the score that corresponds to the regression line) using a score of 99 on the remaining parts of the bar exam is about 19.

The extent to which regression estimates differ from the observed values of the variable we are predicting can be examined by computing a statistic that indicates the magnitude of the difference between the actual values (which are unknown in some cases) and the predicted values for the candidates. This statistic is the *standard error of the estimate* (SEE). The SEE is computed using data from candidates with complete information, for whom we can determine both their actual scores and their predicted scores based on partial information.

Using the regression equation, the SEE, and assumptions about the distribution of observed scores around the regression line (assumed to be a normal distribution), it is possible to estimate the probability that a candidate with a particular predicted score would actually pass or fail if we had complete information and therefore knew the

candidate's bar exam score.⁷ Candidates with predicted scores at the passing score would be expected to have a 50/50 chance of passing. Candidates with predicted scores far above the passing score would have a high probability (close to 1.0) of passing, and therefore a very low probability (close to 0.0) of failing. Candidates with predicted scores far below the passing score would have a very low probability (close to 0.0) of passing, and therefore a very high probability (close to 1.0) of failing.

Candidates with predicted scores below the passing score would have a probability of less than .50 of passing (i.e., less than a 50/50 chance) and a probability of more than .50 of failing if we had complete data. For instance, a candidate with a predicted score that is one SEE below the passing score would have a probability of about .84 of failing and a probability of .16 of passing. That is, about 84% of candidates with this predicted value for the score would be expected to fail if all of the data were known. Similarly, a candidate with a predicted score that is one SEE above the passing score would have a probability of about .84 of passing and a probability of .16 of failing. We can determine the probability that a candidate would pass given his or her estimated bar exam score. As described above, these probabilities can be used in conjunction with a decision rule, or the probability that an estimated bar exam score is at or above the passing score, to make a pass/fail decision.

MAKING A DECISION BASED ON A PREDICTED SCORE

In predicting a score using partial information (i.e., an incomplete set of scores), there are two kinds of correct decisions and two kinds of incorrect decisions, or errors, that can occur in making a decision based on the predicted score.

Candidates are correctly classified if they would have passed if all the data were available and are passed by the decision rule (*true positives*).

Candidates are also correctly classified if they would have failed if all the data were available and are not passed by the decision rule (*true negatives*).

A *false-positive error* is said to occur if a candidate is predicted to pass but would in fact have failed if all of the data were available.

A *false-negative error* is said to occur if a candidate is predicted to fail but would in fact have passed if all of the data were available.

If a candidate's estimated bar exam score is right at the passing score, the candidate would have a 50/50 chance of passing if all of his or her scores were known. If the candidate's predicted score is below the passing score, there is some chance that the candidate would have passed if all of his or her scores were available, and if the candidate's predicted score is above the passing score, there is some chance that he or she would have failed if all of his or her scores were available.

For all scores, any candidate who passes under a particular decision rule is either a true positive or a false positive, and any candidate who fails under the decision rule is either a true negative or a false negative. The decision rule can be used to reduce the number of false negatives or false positives, but not both. For example, a more lenient rule decreases the possibility of false negatives, but increases the chances for false positives. Likewise, a more stringent rule decreases the possibility of false positives, but increases the chance for false negatives.

A HYPOTHETICAL EXAMPLE

To illustrate the three steps in obtaining a pass/fail decision for a candidate missing part of the bar exam, a hypothetical example is provided below in which one candidate is missing a single essay score.

The hypothetical bar examination in this example is made up of 50% MBE, with scores between 0 and 200, and 50% essay questions, with five essay questions with scores between 0 and 40 (and total essay scores between 0 and 200).⁸ Bar exam scores for this hypothetical example are on a 0 to 200 scale and the passing score is 135.

In this example, one candidate is missing a single essay score.⁹ This candidate's performance on the remaining parts of the bar exam is as follows: MBE = 140, essay scores = [missing], 25, 25, 25, 25.

Step 1: Checking the boundaries. The first step is to check the boundaries assuming the candidate receives no credit (i.e., a score of zero) or maximum credit (i.e., a score of 40) on the missing essay.

Assuming a score of zero on the missing essay, this candidate's score would be 120. This candidate would not pass the bar examination with a score of zero on the missing essay. This boundary score was obtained in the following way:

$$.5 \times 140 + .5 \times (0 + 25 + 25 + 25 + 25) = 70 + 50 = 120$$

Assuming a score of 40 on the missing essay, this candidate's score would be 140. This candidate would pass the bar examination with a perfect score on the missing essay. This boundary score was obtained in the following way:

$$.5 \times 140 + .5 \times (40 + 25 + 25 + 25 + 25) = 70 + 70 = 140$$

An unambiguous decision is not obtainable by checking the boundaries because the boundary score in one case leads to a passing score and in the other case leads to a non-passing score.

Therefore, to obtain an estimate of this candidate's performance, we need to predict the score on the missing essay.

Step 2: Predicting the score. To predict this candidate's score on the missing essay, we use the scores from the MBE and the corresponding non-missing essays for candidates with complete information. Specifically, we use linear regression to obtain a linear equation that predicts scores on essay 1 from scores on the MBE and essays 2 through 5. In our hypothetical example, let's say that this process results in a predicted essay score of 26 (and an SEE of 4) for our candidate with the missing essay. Then we obtain an estimated bar exam score of 133 for this candidate, which was calculated as follows:

$$.5 \times 140 + .5 \times (26+25+25+25+25) = 70 + 63 = 133$$

If we take this estimated bar exam score at face value, we will say that the candidate did not pass. However, the predicted score of 26 on the missing essay is a predicted score that is subject to some uncertainty in prediction. Likewise, the estimated bar exam score will be subject to some uncertainty because it uses the predicted score on the missing essay; this candidate's predicted score and actual score (had the missing essay been available) may be different.

Step 3: Defining the decision rule. To account for the uncertainty in prediction, a decision rule should be specified for determining a pass/fail decision. The decision rule can be defined by the probability that the candidate's score would be at or above the passing score on the bar examination.

Table 1 presents a range of possible decision rules, along with the associated decision given our candidate's predicted score of 133. The candidate would pass under the 10% and 25% decision rules and not pass under the 50%, 75%, and 90% decision rules. Of course, the decision rules are not limited to these values; they can take any value between 0% and 100%. The decision rule will need to be determined by the jurisdiction.

Summary. The pass/fail decision for our hypothetical candidate with an estimated bar exam score of 133 depends on the decision rule adopted. Taking

Table 1: Possible Decision Rules and Associated Decisions for a Hypothetical Candidate

Decision Rule*	Decision
10% (lenient)	Pass
25%	Pass
50%	Fail
75%	Fail
90% (stringent)	Fail

*Probability (as a percentage) that the estimated bar exam score of 133 will be greater than or equal to the passing score of 135

the estimated bar exam score at face value (and using the point estimate for the missing essay score), the candidate will not pass the bar exam, but taking into account the uncertainty in the estimated bar exam score, the candidate might pass, depending on how lenient a decision rule is adopted.


CONCLUDING COMMENTS

As a first cut, checking the boundaries may result in an unambiguous pass/fail decision for a candidate missing part of the bar exam. If an unambiguous decision is not obtained, prediction of the score on the missing part to obtain an estimate of the bar exam score is a possible follow-up. However, with prediction, *there is no such thing as a free lunch*. Predicted scores are estimates and are subject to uncertainty. The predicted score is the score expected on the missing part based on scores for the non-missing parts of the bar exam, but it is not necessarily the score that the candidate would have received on the missing part if that part were not missing.

Using the predicted scores and making some assumptions about the distribution of uncertainty in the predicted scores, we can obtain an estimated probability that the bar exam score a candidate would have obtained is at or above the passing score. By defining a decision rule as a probability, we can determine whether or not a candidate passes the bar exam, given a candidate's predicted score on the missing part of the exam. Decision rules are ultimately policy decisions and involve trade-offs between stringency and leniency.

The process of predicting a score on the missing part, estimating the bar examination score, and estimating a pass/fail decision is the last resort when part of the bar exam is missing. While the general process for obtaining an estimated pass/fail decision

is likely to be the same in all cases, each situation is unique and should be considered carefully before endeavoring to predict a score on a missing part. In most cases, a jurisdiction should consider seeking assistance from a statistical consultant or NCBE when a decision has been made to pursue a predicted score for a candidate missing part of the bar exam.

Making a pass/fail decision for a candidate missing part of the bar exam can involve tough choices. A jurisdiction must walk a fine line between giving the candidate the benefit of the doubt and protecting the public. 

ENDNOTES

1. Another defensible option for checking the boundaries is to substitute the lowest score observed for all other candidates on the missing part and the highest score observed for all other candidates on the missing part of the exam. While a perfect score and a zero score are hypothetically possible, often these scores are not observed on a particular administration of the exam. If the minimum observed score on the missing part is greater than zero, this approach to checking the boundaries will result in a more lenient pass/fail decision for candidates who would be identified as unambiguously passing the exam. If the largest observed score is less than the maximum possible score on the missing part, this approach to checking the boundaries will result in a more stringent pass/fail decision for candidates who would be identified as unambiguously failing the exam.
2. It is possible to use other sorts of regression methods or other types of statistical models to predict a score on the missing part, but linear regression is the most commonly used regression method in prediction contexts such as this one.
3. This method of obtaining the regression line is referred to as the *least squares* method.
4. The essay is on a 0–40 scale, which is used in the hypothetical example presented later. See note 8.
5. This hypothetical plot and hypothetical regression line are oversimplifications of the regression line that would be obtained in an actual situation when a missing part is being predicted because in such situations, the typical analysis involves entering the scores for individual components of the bar exam (e.g., MBE and each individual essay score) into the regression equation separately using what is referred to as *multiple regression*. In such an analysis, the associations between individual non-missing components and the missing component are taken into account when estimating a

regression line. Multiple regression is difficult to present in a simple way using a single plot, so for purposes of illustration, multiple regression is glossed over a bit in this article in order to keep the example simple and easier to conceptualize; the hypothetical example here using scores on the non-missing parts to predict a missing essay score is illustrated using *bivariate regression* (which involves two variables and can be easily plotted), which will generally be less accurate at predicting the score on the missing part than multiple regression.

6. The relationship between scores on essay 1 and bar exam scores without essay 1 in this example is strong, with a correlation of .84.
7. The probability of passing given a particular passing score, an SEE, and an estimated bar exam score is obtained by identifying the area less than or equal to the estimated bar exam score under a normal curve with mean defined by the passing score and standard deviation defined by the SEE.
8. This essay scale differs from the typical six-point scale used by NCBE (and many jurisdictions) for the MEE and MPT. The 0–40 scale was used for the essays in this example for convenience and to avoid the added complexity of scaling essay scores to the MBE and appropriately weighting bar exam parts; with the 0–40 scale, it is assumed that the essay scores have been appropriately scaled to the MBE, and the essays can be more easily weighted 50% of the bar exam. For more information on scaling essay scores to the MBE, see Case, Susan M., *The Testing Column: Demystifying Scaling to the MBE: How'd You Do That?* 74 THE BAR EXAMINER 2:45 (May 2005). For more information on weighting examination components, see Case, Susan M., *The Testing Column: Best Practices with Weighting Examination Components*, 77 THE BAR EXAMINER 1:43 (February 2008).
9. Of course, a candidate could be missing the score on any essay (or multiple essays) or any part of the bar exam.



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